Continuous learning in dendritic cortical microcircuits using Lagrangian mechanics

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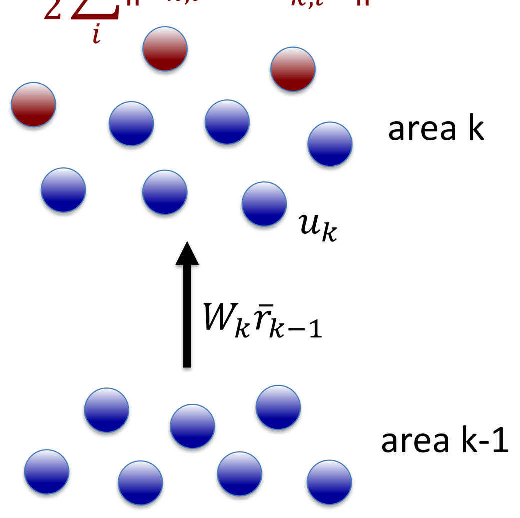
1. Energy-based models

Energy function encodes network:

$$E = \frac{1}{2}\sum_{k=1}^{N} \underbrace{\|u_k - W_k \bar{r}_{k-1}\|^2}_{\text{prediction error}} + \beta \underbrace{C}_{\text{cost function}}$$

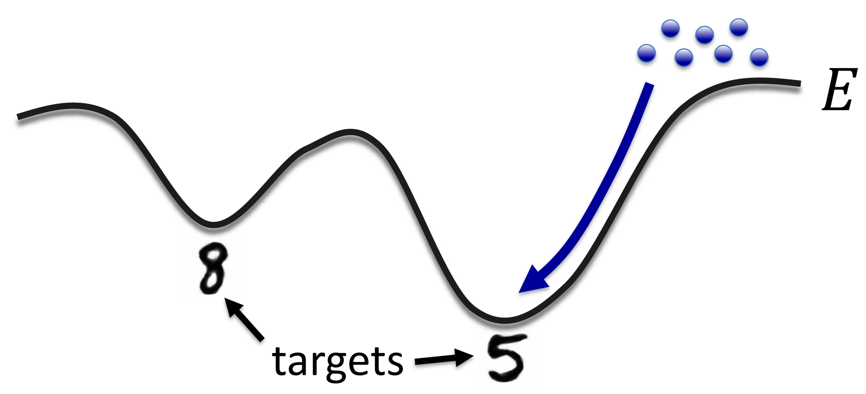
$$C = \frac{1}{2}\sum_i \|u_{k,i} - u_{k,i}^{trgt}\|^2$$



Standard -- dynamics from gradient descent:

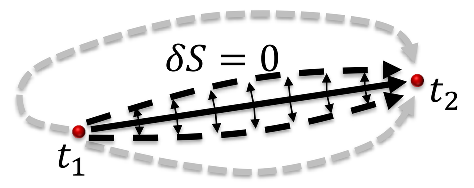
$$\dot{u} \sim -\nabla_u E$$

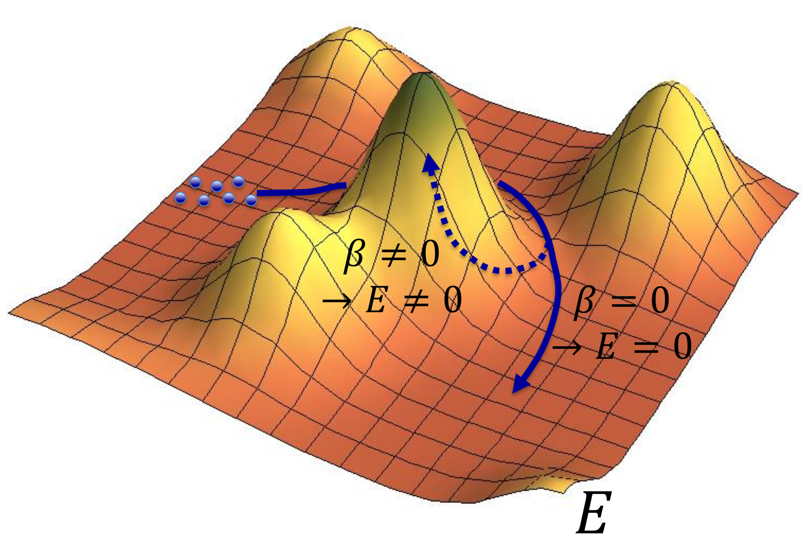
$$\dot{W} \sim -\nabla_W E$$



Here -- dynamics from Lagrangian mechanics:

$$\underbrace{S}_{\text{action}} = \int_{t_1}^{t_2} dt \underbrace{L(\tilde{u}, \dot{\tilde{u}})}_{\text{Lagrangian}}$$





$$\frac{\partial L}{\partial \tilde{u}} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\tilde{u}}} = 0 \longrightarrow$$

no phases, continuous learning possible

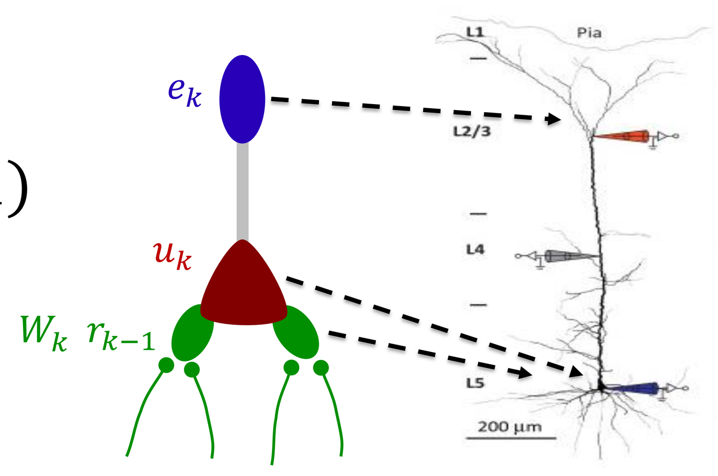
2. From Euler-Lagrange to neurons

Use future discounted voltage:

$$\tilde{u}(t) = \frac{1}{\tau}\int_t^\infty dt' u(t') e^{-\frac{t'-t}{\tau}} \xrightarrow{L(\tilde{u},\dot{\tilde{u}}) = -E(u)} \frac{\partial L}{\partial \tilde{u}} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\tilde{u}}} = 0$$

Resulting neuronal dynamics:

$$\underbrace{\tau \dot{u}_k = -u_k}_{\text{somatic integration}} + \underbrace{W_k r_{k-1} + e_k}_{\text{basal and apical compartment}} \quad (1)$$

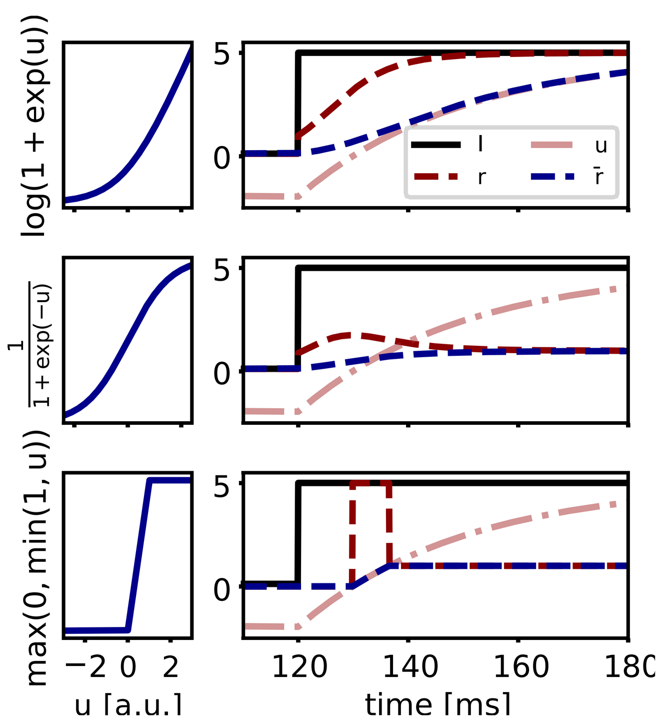


Advanced neuronal response:

$$r_k = \underbrace{\bar{r}_k + \tau \dot{\bar{r}}_k}_{\text{phase-advance of } \bar{r}_k = \varphi(u_k)}$$

$$\approx \bar{r}_k(t + \tau)$$

$$\approx \underbrace{m_\infty^3 h(u, \dot{u})}_{\text{sodium gating of HH neurons}}$$



3. Learning with dendrites

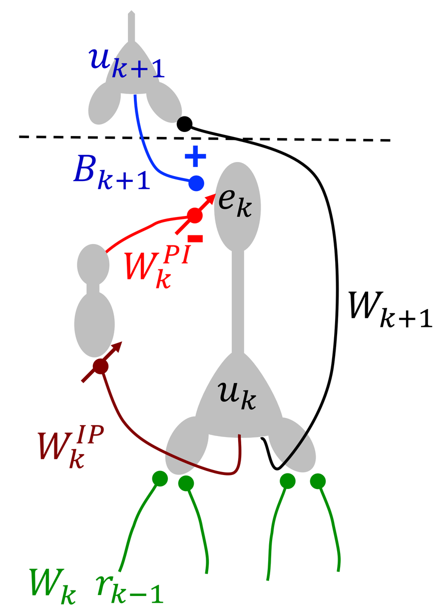
Prediction error encoded in apical dendrites:

$$\bar{e}_k = \bar{r}'_k \cdot W_{k+1}^T (u_{k+1} - W_{k+1}\bar{r}_k)$$

$$\sim W_{k+1}^T u_{k+1} - W_{k+1}^T W_{k+1} \bar{r}_k$$

$$\sim \underbrace{B_{k+1} u_{k+1}}_{\text{top-down feedback}} - \underbrace{W_k^{PI} W_k^{IP} \bar{r}_k}_{\text{bottom-up prediction}}$$

⟶ **nudges u_k away from basal voltage**



Basal prediction of soma drives plasticity:

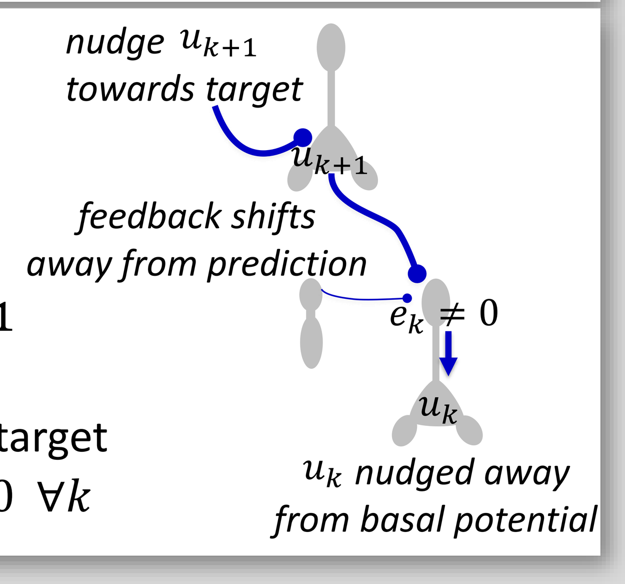
$$\dot{W} \sim -\nabla_W E \longrightarrow \dot{W}_k \sim \underbrace{(u_k - W_k \bar{r}_{k-1})}_{\text{Urbanczik-Senn (basal prediction of soma)}} \bar{r}_{k-1}^T \quad (2)$$

Backpropagation of errors:

combining (1) and (2): $\dot{W}_k \sim \bar{e}_k \bar{r}_{k-1}^T$

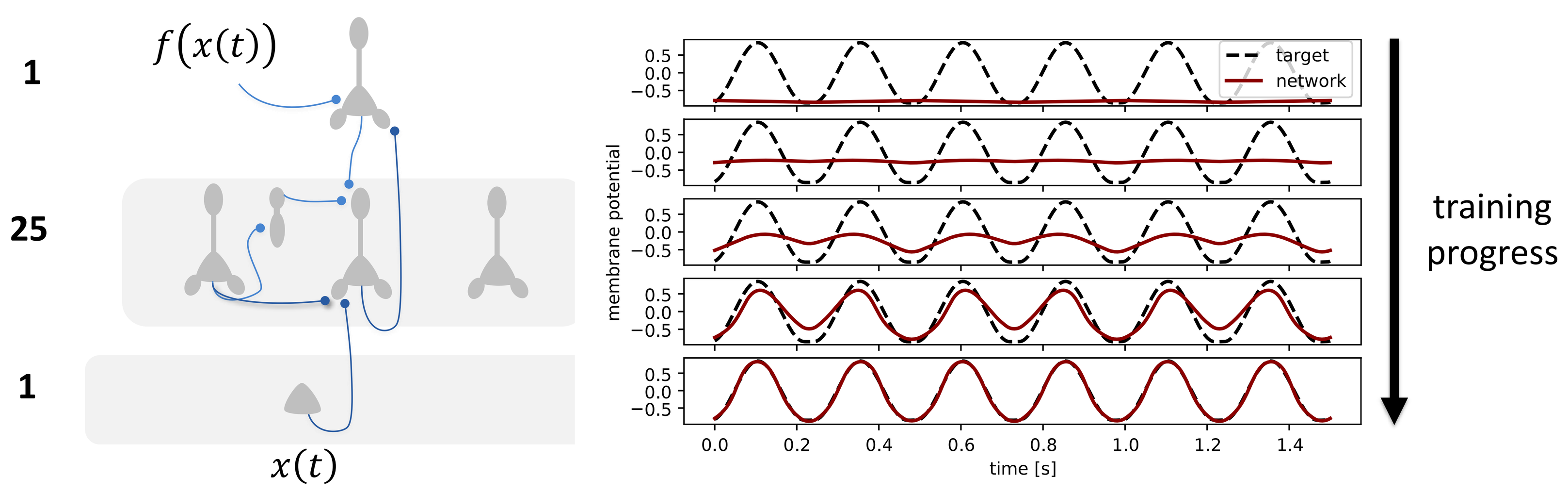
with $\bar{e}_k = \bar{r}'_k \cdot W_{k+1}^T \bar{e}_{k+1}$

- **errors** introduced by **nudging** neurons towards target
- no nudging ⟶ no errors ⟶ $\dot{W}_k = 0 \;\; \forall k$

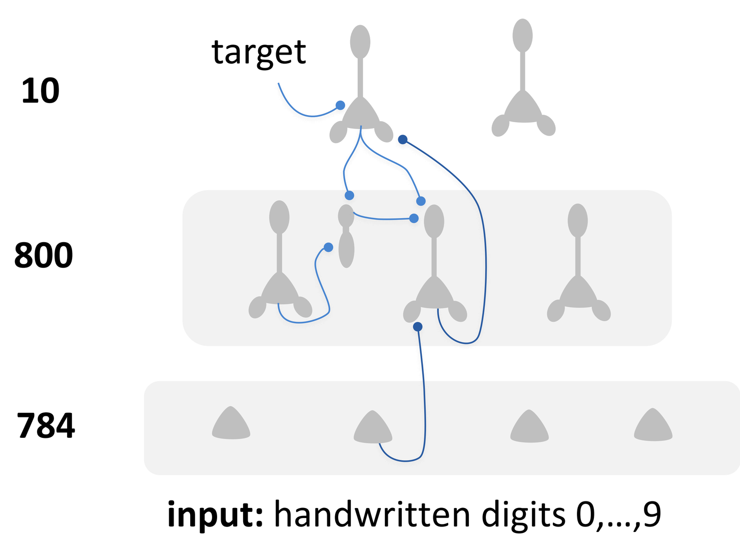


4. Time-continuous learning

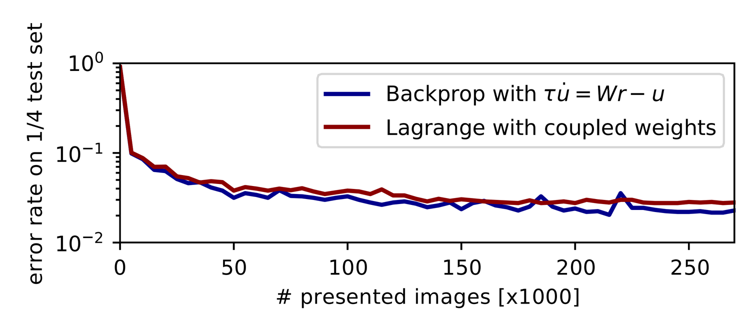
Learning a simple nonlinear continuous mapping:



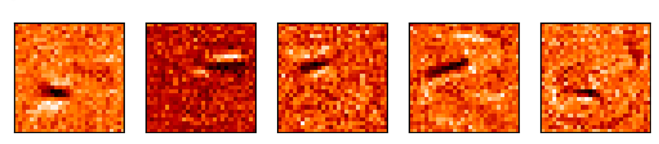
Learning handwritten digits:



Performance:



Receptive fields:



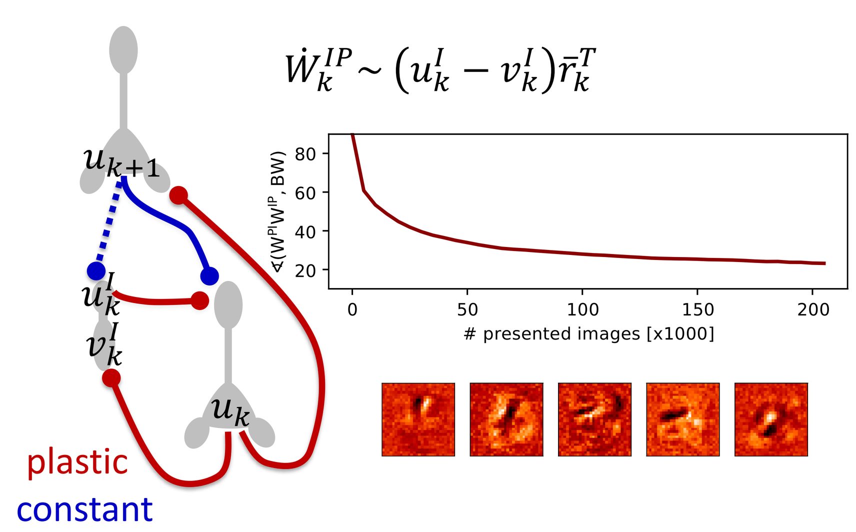
Learning lateral weights:

from theory: coupled forward and backward weights

deviation: weights align during training *(work in progress)*

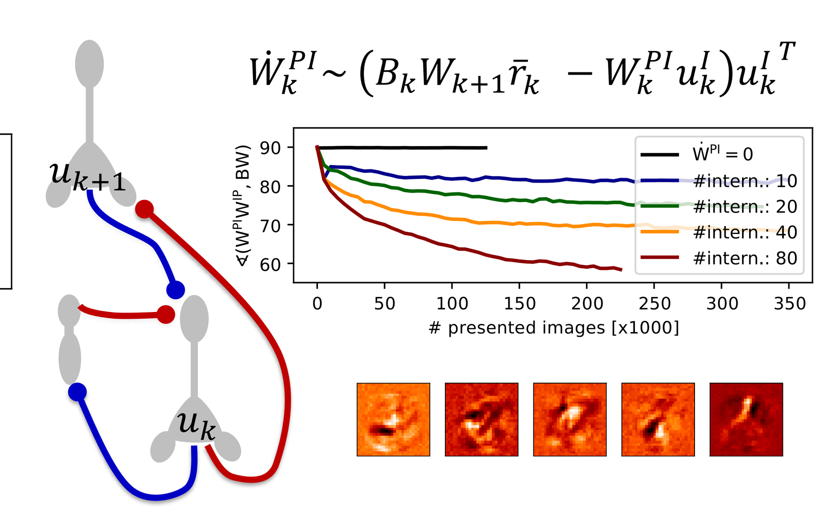
nudging of interneurons:

$$\dot{W}_k^{IP} \sim (u_k^I - v_k^I)\bar{r}_k^T$$



no nudging of interneurons:

$$\dot{W}_k^{PI} \sim (B_k W_{k+1}\bar{r}_k - W_k^{PI} u_k^I) u_k^{I\,T}$$



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